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Low-Thrust Flyby Guidance and an Extended Optimal Spacing Rule

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A new extended spacing law governing midcourse correction points is given for electrically propelled interplanetary missions. The use of electric propulsion is inappropriate for guidance of flyby missions because little time is available to guide the spacecraft, especially when it is within close range of the target. Therefore, some chemical fuel is inevitably required onboard the spacecraft. The problem treated here is to determine how much chemical fuel is required to cancel out the potential electric and impulsive correction errors. A generalized spacing rule that determines the optimal correction points is derived. In addition, a simple analytical fuel estimate is presented. Numerical illustrations include a Monte Carlo simulation as well as the return flight from the asteroid back to Earth with rigorous ephemerides and time system, demonstrating the newly developed spacing rule.

Nomenclature

a_{ep}	= acceleration by the electric propulsion
c	= coasting
$E[\,]$	= expectation operator
Eps_{xf}	= specified terminal miss distance allowance
j	= time node
k	= correction epochs
n	= number of correction nodes or the number of time index
S	= total velocity increment accelerated by the electric propulsion
T	= flight period to the encounter
T'	= centroid time or end time of the coasting arc
α	= flight time ratio
ΔV	= impulsive correction velocity increment
Δx_f	= terminal miss distance
$\delta(\Delta V)$	= impulsive correction mechanization error
$\delta\phi$	= electric correction mechanization error
ε	= velocity navigation error (standard deviation)
η	= proportional coefficient of the sensitivity matrix norm to the flight time
λ	= combined parameter representing total velocity error (standard deviation)
ν	= characteristic time defining the navigation governed spacing boundary
ρ	= position navigation error (standard deviation)
τ	= representative time required for the electric propulsion to compensate for the navigation error
Φ	= sensitivity matrix of the velocity deviation to the terminal position
ϕ	= electric correction margin
0	= total flight period or the total correction margin
^	= total electric correction margin

Introduction

RECENT prospective interplanetary mission designs include the use of electric propulsion to reduce the amount of chemical fuel required and to enhance payload capability.[1] Flyby missions to small celestial bodies, such as asteroids and comets, have been intensively investigated trying to include the advantages of electric propulsion. The significant factor in such mission analyses is the amount of fuel required to compensate for the potential correction errors in chemical as well as in electric maneuvers. Fundamentally, most corrections may be carried out via the electric propulsion system, and almost no chemical fuel may be required for guidance because the electric propulsion is capable of accelerating the spacecraft. However, this discussion is only partly true. Both the acceleration direction error and potential efficiency degradation in electric propulsion have to be taken into account, leading to the need for a tremendous amount of chemical fuel onboard. In particular, electric propulsion is inappropriate for guidance of flyby missions that require a certain relative terminal velocity with respect to the target bodies. Because of the inherent time constraints and considerations of the mission, little time is available for electric maneuvers to guide the spacecraft to encounter the target, especially when the spacecraft is within close range of the target. Therefore, to compensate, some chemical fuel is inevitably required onboard the spacecraft. The discussion here concentrates on how much chemical fuel is needed for missions propelled by an electric propulsion system.

Historically speaking, this type of problem was dealt with by Breakwell,[2,3] who developed the so-called spacing rule, taking the impulsive correction error into account statistically. Other intensive studies were made by Lawden and Long[4] and Lawden,[5,6] who also investigated the guidance and correction problem. Ghaffari[7] applied the spacing schemes to the trans-Jupiter case and established a treatment of multiple midcourse maneuvers where ballistic problems are concerned. Although Battin[8,9] investigated the problem and proposed the variance ratio method and Pfeiffer[10] proposed the dynamic programming approach, the spacing ratio method has been frequently used so that the total required fuel amount can be minimized.[11,12] Therefore, the first question is whether the ordinary spacing law can be applied to electrically propelled missions. Previous studies on the guidance of low-thrust missions were dedicated only to steering problems[13] and little attention has been focused on chemical correction problems. This paper shows mathematically that the answer to the question just raised is negative; a different spacing rule has to be developed. This paper presents the extended spacing law that defines the optimal correction interval from any point to the flyby time. A practical analytical estimate for the chemical fuel amount is also given. Another novel product, not directly related to the electric propulsion problem, is a spacing rule governing proximity guidance within 10–20 days prior to the encounter. This rule is driven primarily by the navigation error statistics.

Numerical illustrations are shown for both the simplified flight example demonstrated with a Monte Carlo simulation and the return flight path from an asteroid to the Earth propelled via an electric propulsion system. The first example stresses the statistical discussion employed in this paper, whereas the second example demonstrates an actual mission application with the rigorous ephemerides and time system. For the cases with varied acceleration and coasting arcs, generalized results of the extended spacing laws are also given

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in the Appendices. They can be applied to most of the missions propelled by electric propulsion.

New Extended Spacing Law

Usually the orbit synthesis via electric propulsion is carried out so that the minimum velocity increment is accomplished and the coasting arc period can be expanded as much as possible. This synthesis mechanism is similar to that of the multilink manipulator, in which the total manipulator length corresponds to the total velocity increment. When full optimization is accomplished, those links are fully extended. Consequently, if the propulsion system does not perform as planned, use of a coasting period may be the only solution. Additionally, when all of the links are fully extended without any coasting arc, the orbit sequence noticeably lacks a correction margin. Consequently, two cases will be considered: 1) where the coasting period is fully replaced with the powered period and 2) where, because of onboard fuel restrictions, the maximum number of powered arcs is restricted and coasting arcs exist. It is assumed that in both cases any synthesized orbit sequence has not fully extended its velocity increment, but has some correction margins designated by $\delta\phi_k$, for the kth subarc.

Usually a sensitivity matrix is required for assessing how the maneuvers effect the terminal miss distance. Although it is dependent on time, the norm of the sensitivity matrix is almost linear and can be approximated as proportional to the flight time from the maneuver point to the encounter. Evidence of this approximation is shown later in the numerical examples.

The terminal miss distance at the $(k+1)$th correction epoch is expressed as follows:

$$E\left[\Delta x_{f,k+1}^2\right] = \eta^2\left(\tfrac{1}{2}a_{\mathrm{ep}}T_k^2 - \tfrac{1}{2}a_{\mathrm{ep}}T_{k+1}^2\right)^2 E\left[\delta\phi_k^2\right] + \eta^2 T_k^2 E\left[\delta(\Delta V)_k^2\right] + \eta^2 T_k^2 E\left[\varepsilon_k^2\right] + E\left[\rho_k^2\right] \tag{1}$$

Note that T_k here stands for the flight period from the kth epoch to the terminal flyby time and η is the proportional coefficient, with which the sensitivity is approximated as ηT_k. In the gravity-free uniform field, η is identical to unity. Although η depends on and varies along with the trajectory, a certain η can represent the sensitivity. It is written here in the stochastic probability form, root sum square (rss), assuming each quantity is the standard deviation. $\Delta x_f/\eta$ and ρ/η are redefined as Δx_f and ρ, and when the terms on both sides of Eq. (1) are replaced by the scalar variables, the largest terminal distance at the $(k+1)$th epoch is expressed as follows:

$$\Delta x_{f,k+1} = \left(\tfrac{1}{2}a_{\mathrm{ep}}\delta\phi_k T_k^2 - \tfrac{1}{2}a_{\mathrm{ep}}\delta\phi_k T_{k+1}^2\right) + T_k\delta(\Delta V)_k + T_k\varepsilon_k + \rho_k \tag{2}$$

The following shows the min–max approach taken here that minimizes the worst correction amount corresponding to the maximum terminal distance. The first term indicates the terminal miss distance derived from the unexpected fluctuation in the electric propulsion during the period from the kth to the $(k+1)$th epoch. This quantity is characterized via $\delta\phi_k$, representing errors such as the acceleration direction error and the degraded propulsion efficiency. The second term stands for the contribution from the mechanization error at the kth chemical maneuver. Even if the correction is performed as calculated at the kth point, the calculation must be based on the navigation information and a certain amount of navigation error that inevitably takes place and contributes to the terminal miss distance, expressed in the third and fourth terms of Eq. (2). The question is whether the left-hand side of Eq. (2) is what should be equal to the $(k+1)$th maneuver capability: $\frac{1}{2}a_{\mathrm{ep}}\delta\phi_{k+1}T_{k+1}^2 + T_{k+1}\Delta V_{k+1}$.

The answer is slightly different. When the correction is performed, the anticipated correction amount is based on the $(k+1)$th navigation information, and what is calculated for the correction may exceed the amount. As a result, the terminal miss distance is connected to the correction strategy at the $(k+1)$th epoch as follows:

$$\tfrac{1}{2}a_{\mathrm{ep}}\delta\phi_{k+1}T_{k+1}^2 + T_{k+1}\Delta V_{k+1} = \Delta x_{f,k+1} + T_{k+1}\varepsilon_{k+1} + \rho_{k+1} \tag{3}$$

This equation is solved for the $(k+1)$th chemical correction amount as

$$\Delta V_{k+1} = \frac{T_k}{T_{k+1}}\lambda_k + \frac{1}{T_{k+1}}(\rho_k + \rho_{k+1}) + \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k\frac{T_k^2 - T_{k+1}^2}{T_{k+1}} - \frac{1}{2}a_{\mathrm{ep}}\phi_{k+1}T_{k+1} + \varepsilon_{k+1} \tag{4}$$

The velocity error at the kth point λ_k is

$$\lambda_k = \delta(\Delta V)_k + \varepsilon_k \tag{5}$$

Note that the worst correction amount may be accumulated in terms of the navigation errors. The terminal miss distance, however, is always governed by Eq. (2) and the navigation errors actually neither accumulate nor affect the terminal miss distance. Equation (2) shows the contribution of the kth correction strategy whose estimate exceeds the exact amount as in Eqs. (3) and (4).

The final correction point is uniquely determined so that the specified terminal error is guaranteed. The following relation holds:

$$\Delta x_{f,n+1} = \tfrac{1}{2}a_{\mathrm{ep}}\delta\phi_n T_n^2 + T_n\delta(\Delta V)_n + T_n\varepsilon_n + \rho_n \tag{6}$$

Assume that it will be within the specified admissible range

$$\Delta x_{f,n+1} \le Eps_{xf} \tag{7}$$

Then the final correction epoch T_n is obtained as

$$T_n = \frac{\sqrt{\lambda_n^2 - 2a_{\mathrm{ep}}\delta\phi_n(\rho_n - Eps_{xf})} - \lambda_n}{a_{\mathrm{ep}}\delta\phi_n} \cong \frac{Eps_{xf} - \rho_n}{\lambda_n} \tag{8}$$

The primary focus is to minimize the worst correction velocity amount, which is the sum of the chemical correction ΔV_k,

$$\Delta V_{\mathrm{total}} = \sum_{k=1}^{n}\Delta V_k = \sum_{k=2}^{n}\left\{\frac{T_{k-1}}{T_k}\lambda_{k-1} + \frac{1}{T_k}(\rho_{k-1} + \rho_k) + \frac{1}{2}a_{\mathrm{ep}}\delta\phi_{k-1}\frac{T_{k-1}^2 - T_k^2}{T_k}\right\} + \Delta V_1 - \sum_{k=1}^{n}\frac{1}{2}a_{\mathrm{ep}}\phi_k T_k \tag{9}$$

The last term ε_k in Eq. (4) does not affect the total correction $\Delta V_{\mathrm{total}}$ via T_k, and the correction point is determined by examining Eq. (9). Therefore, ε_k is neglected here. Note that the electric propulsion correction is not of concern here since the powered arcs are frozen in this analysis as noted at the beginning of this section. Besides, as already noted, the electric correction margin is restricted with an upper limit. This mechanism is easily understood in view of the fact that each arc has a fixed link length corresponding to the velocity increment. The relation may be written as

$$T_0\hat{\phi} = \sum_{k=1}^{n}T_k\phi_k \tag{10}$$

The discussion here assumes that the electric correction margins are already exhausted at each correction point to investigate the worst-case chemical fuel amount required onboard. Consequently, the total chemical correction velocity increment is expressed as follows:

$$\Delta V_{\mathrm{total}} = \sum_{k=2}^{n}\left\{\frac{T_{k-1}}{T_k}\lambda_{k-1} + \frac{1}{T_k}(\rho_{k-1} + \rho_k) + \frac{1}{2}a_{\mathrm{ep}}\delta\phi_{k-1}\frac{T_{k-1}^2 - T_k^2}{T_k}\right\} + \Delta V_1 - \frac{1}{2}a_{\mathrm{ep}}\hat{\phi}T_0 \tag{11}$$

The optimal kth correction point is determined by differentiating the Eq. (11) with respect to T_k:

$$\frac{\partial\Delta V_{\mathrm{total}}}{\partial T_j} = \frac{\partial}{\partial T_j}\left\{\frac{1}{T_j}(\rho_{j-1} + \rho_j)\right\} + \frac{\partial}{\partial T_j}\left\{\frac{T_{j-1}}{T_j}\lambda_{j-1} + \frac{T_j}{T_{j+1}}\lambda_j\right\} + \frac{1}{2}a_{\mathrm{ep}}\frac{\partial}{\partial T_j}\left\{\delta\phi_{j-1}\frac{T_{j-1}^2 - T_j^2}{T_j} + \delta\phi_j\frac{T_j^2 - T_{j+1}^2}{T_{j+1}}\right\} \tag{12}$$

which gives

$$\frac{T_k}{T_{k+1}} = \frac{T_{k-1}}{T_k}\frac{\lambda_{k-1}}{\lambda_k} + \frac{1}{T_k}\frac{\rho_{k-1}+\rho_k}{\lambda_k} + \frac{1}{2}\frac{a_{\text{ep}}\delta\phi_{k-1}}{\lambda_k}\frac{T_{k-1}^2+T_k^2}{T_k} - \frac{a_{\text{ep}}\delta\phi_k}{\lambda_k}\frac{T_k^2}{T_{k+1}} \tag{13}$$

Finally, if $\delta(\Delta V)_k$, $\delta\phi_k$, ε_k, and ρ_k are regarded as constants, the following advanced spacing rule is derived:

$$\frac{T_k}{T_{k+1}} = \frac{T_{k-1}}{T_k} + \frac{1}{T_k}\nu + \frac{\delta\phi}{2\tau}T_k\left(1 + \frac{T_{k-1}^2}{T_k^2} - 2\frac{T_k}{T_{k+1}}\right)$$
$$\nu = 2\rho/\lambda, \qquad \tau = \lambda/a_{\text{ep}} \tag{14}$$

where τ indicates the representative time required for compensation of the velocity error via electric propulsion. Here ν defines the flight time boundary within which the navigation error contribution dominates in terms of the terminal miss distance. The relation [Eq. (14)] is newly derived and characteristic of the fact that both impulsive and low-thrust correction methods are combined at the same time. The first term on the right-hand side of Eq. (14) represents the significance of the impulsive maneuver error, and the second and third terms indicate the effect of navigation and electric correction maneuver errors, respectively.

Flight Regime Dependent Spacing Rule and Correction Velocity

As shown here, the new extended spacing rule in Eq. (14) will have to be altered depending on the flight regime. Conventional and contemporary deep space navigation[14] states that the most probable values for parameters in the spacing rule law are as shown in Table 1. The figures given in Table 1 refer to state-of-the-art technology, where ground-based radio metric navigation [range and range rate (R&RR) technique] is used. Note that position navigation error is almost proportional to the distance from the Earth and may be measured by the angular resolution. The contemporary R&RR technique is well within 1 μrad and the values listed in Table 1 are for a distance of 1 AU from the Earth. Note that $\delta\phi$ indicates not only the acceleration direction error but the propulsion performance error as well.

With the figures from Table 1, ν and $2\tau/\delta\phi$ are calculated as 21 and 16 days, respectively. The magnitude of the terms on the right-hand side of Eq. (14) is examined and tabulated in Table 2, in which T_k is varied. Note that the third term, the electric correction error effect, dominates in the range of more than 100 days in T_k, regimes 1 and 2. Therefore, in these flight regimes, only the third term need be considered, altering Eq. (14) to the following degenerated form:

$$T_{k-1}^2/T_k^2 = 2(T_k/T_{k+1}) - 1 \tag{15}$$

Table 1 Typical navigation and guidance parameters

$\rho = 1 \times 10^5$	Position navigation error in interplanetary field, m
$\varepsilon = 0.1$	Velocity navigation error in interplanetary field, m/s
$a_{\text{ep}} = 1 \times 10^{-4}$ to 1×10^{-5}	Typical electric propulsion acceleration, m/s^2
$\delta\phi = 0.1(2 \times 10^{-3})$	Electric correction maneuver error, deg (rad)
$\delta(\Delta V) = 0.01$	Chemical correction maneuver error, m/s

Table 2 Flight regime and contribution to spacing

Regime	T_k, days	First term	Second term	Third term
1	1,000	0.1–1	0.01	10–100
2	100	0.1–1	0.1	1–10
3	10	0.1–1	1	0.1–1
4	1	0.1–1	10	0.01–0.1

The rule in Eq. (15) is independent of any navigation and impulsive parameters and should be the characteristic spacing rule for electric propulsion. Considering that almost all of the cruising flights in the interplanetary field belong to regimes 1 and 2, Eq. (15) should be the unique spacing rule for electrically propelled missions.

When the remaining flight time is less than 100 days, the contributions from the second and third terms become comparable. Immediately the spacing rule is governed only by the navigation effect. Neglecting the third term in Eq. (14), an alternative rule is obtained:

$$T_k/T_{k+1} = (T_{k-1}/T_k) + (1/T_k)\nu \tag{16}$$

This rule may be applied in regimes 3 and 4. In regime 4, Eq. (16) is further reduced to the following form:

$$T_k/T_{k+1} = (1/T_k)\nu \tag{17}$$

Here ν indicates the boundary time within which the spacing rule changes to Eq. (17). What is to be stressed here is that the ordinary spacing rule never appears in electrically propelled cruising in the interplanetary field. If electric propulsion is excluded, in other words, if the third term is removed from Eq. (14) by equating $\delta\phi$ to zero, the situation changes drastically. As Table 2 shows, the first term contribution is significant in regimes 1 and 2, which covers almost all of the flight period in interplanetary missions. If no electric propulsion is used, an alternative rule is obtained as follows:

$$T_k/T_{k+1} = T_{k-1}/T_k \tag{18}$$

This relation is what Breakwell[3] derived in the past and is the ordinary spacing rule. As discussed here, Eq. (14) is the generalized spacing rule that covers all flight regimes. A new rule, Eq. (15), is found to be the unique spacing law throughout most of the flight period, if the mission is employing electric propulsion.

Returning to the fundamental problem raised in the Introduction, an analytical evaluation of the chemical correction velocity increment is sought. For this purpose, the optimal spacing Eq. (14) is incorporated in Eq. (11), which gives

$$\Delta V_{\text{total}} = a_{\text{ep}}\tau\sum_{k=2}^{n}\left\{\frac{T_k}{T_{k+1}} - \frac{\delta\phi}{\tau}T_k\left(1 - \frac{T_k}{T_{k+1}}\right)\right\} + \Delta V_1 - \frac{1}{2}a_{\text{ep}}\hat{\phi}T_0 + n\varepsilon \tag{19}$$

The last term indicates the contribution represented by the last term in Eq. (4). Assume that the number of corrections is large and that T_k/T_{k+1} gets close to unity as follows:

$$T_k/T_{k+1} = 1 + \alpha_k \tag{20}$$

Since α_k is small, Eq. (19) is approximated and reduced to

$$\Delta V_{\text{total}} \cong a_{\text{ep}}\sum_{k=2}^{n}\{\tau + \delta\phi\alpha_k T_k\} + \Delta V_1 - \frac{1}{2}a_{\text{ep}}\hat{\phi}T_0 + n\varepsilon \tag{21}$$

Referring to the new spacing law (15), the following relation is derived:

$$\alpha_k = \alpha_{k+1} + \tfrac{1}{2}\alpha_{k+1}^2 \tag{22}$$

and in view of the relations

$$T_k = T_{k+1} + \alpha_k T_{k+1} = T_{k+1} + \alpha_{k+1}T_{k+1} + \tfrac{1}{2}\alpha_{k+1}^2 T_{k+1}$$
$$\alpha_k T_k = T_{k-1} - T_k - \tfrac{1}{2}\alpha_k^2 T_k \cong T_{k-1} - T_k \tag{23}$$

the total chemical velocity increment is obtained as follows:

$$\Delta V_{\text{total}} \cong a_{\text{ep}}\sum_{k=2}^{n}[\tau + \delta\phi(T_{k-1} - T_k)] + \Delta V_1 - \frac{1}{2}a_{\text{ep}}\hat{\phi}T_0 + n\varepsilon$$
$$= a_{\text{ep}}[(n-1)\tau + \delta\phi(T_1 - T_n)] + \Delta V_1 - \frac{1}{2}a_{\text{ep}}\hat{\phi}T_0 + n\varepsilon \tag{24}$$

In the particular case where the electric correction margin is exhausted and the first correction is excluded, the following equation results:

$$\Delta V_{\text{total}} = (n-1)\lambda + a_{\text{ep}}\delta\phi(T_1 - T_n) + n\varepsilon \quad (25)$$

The first correction may be removed from discussion because the spacing law does not apply to the first correction, primarily because the first correction is dedicated to correcting injection errors requiring a tremendously large amount of velocity increment. The first and the third term in Eq. (25) relate to the ordinary spacing rule derived from the chemical correction error, whereas the second term indicates that the total electric correction error has to be compensated for via chemical correction. That is compatible with the heuristic interpretation.

The preceding discussion concerning the fundamental spacing equation assumed an acceleration level and no coasting arc. In some practical applications, however, these assumptions are not valid and a more generalized relation should be developed. The theoretical process so far can be directly applied to those practical cases, and the mathematical manipulations are shown later in the Appendices. A summary of the results in the Appendices follows. When the acceleration level is varied during the flight, the newly developed spacing rule, Eq. (15), remains valid, as it appears in Eq. (A6). In the cases with a coasting period, the alternative spacing rule corresponding to Eq. (15) is easily reformulated as Eq. (B7). In combination with Eq. (A6), Eq. (B7) provides the most generalized spacing rule, which is versatile for use by most of the missions utilizing electric propulsion.

Numerical Illustration

First evidence is shown here for approximating the sensitivity matrix as proportional to the flight time to the encounter, as in Eqs. (1) and (2). Figure 1 shows the so-called spectrum radii that are the square roots of the largest eigenvalues for matrix $\Phi^T\Phi$ where Φ denotes the sensitivity matrix of the velocity deviation to the terminal position. Three distinct plots are drawn, corresponding to the distinct ellipses with different aphelion distances. Φ stands for the sensitivity from the perihelion to the specified point. Figure 2 directly shows the proportional coefficient η in the preceding section. These two figures suggest that a certain η exists, with which the sensitivity is approximated as ηT_k. Practically, η has to be examined by integrating the trajectory, and Δx_f and ρ have to be altered appropriately.

Two numerical simulations are presented here. The first example, a Monte Carlo simulation statistically illustrating which spacing rule is optimal, demonstrates the discussion in this paper. The mathematical model here is the uniform gravity-free field. Three corrections are assumed, among which the first and third epochs are frozen. The problem is simplified by finding the optimal second correction point. The electrical correction margin is assumed to be exhausted at the first correction, and no margin is expected at the second and third corrections. Four stochastic quantities, position and velocity navigation errors and impulsive and electrical correction mechanization errors, are generated based on the Gaussian random model here. Three distinct flight periods were tested and each simulation attempted 100,000 cases. The flight periods tested are 10, 30, and 100 days. The numerical data are summarized in Table 3.

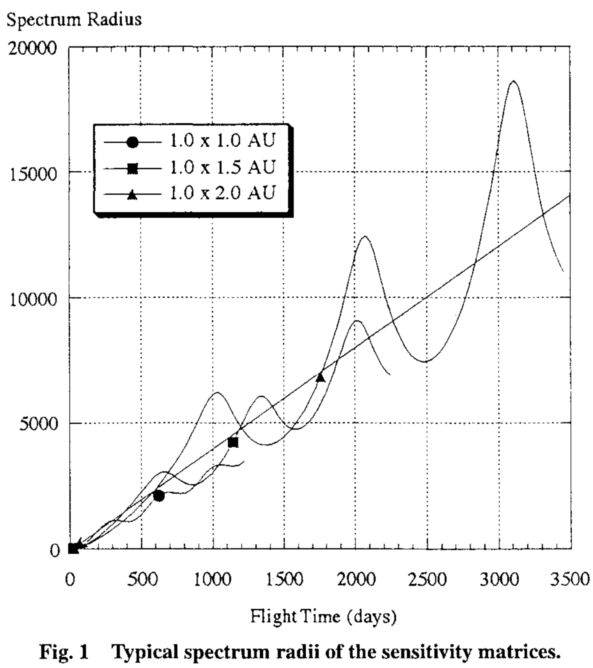

Fig. 1 Typical spectrum radii of the sensitivity matrices.

Table 3 Numerical simulation data

ρ	1×10^5 m
ε	0.1 m/s
a_{ep}	1×10^{-4} m/s^2
ϕ	10 deg
$\delta\phi$	0.1 deg
$\delta(\Delta V)$	0.01 m/s
Eps_{xf}	150 km

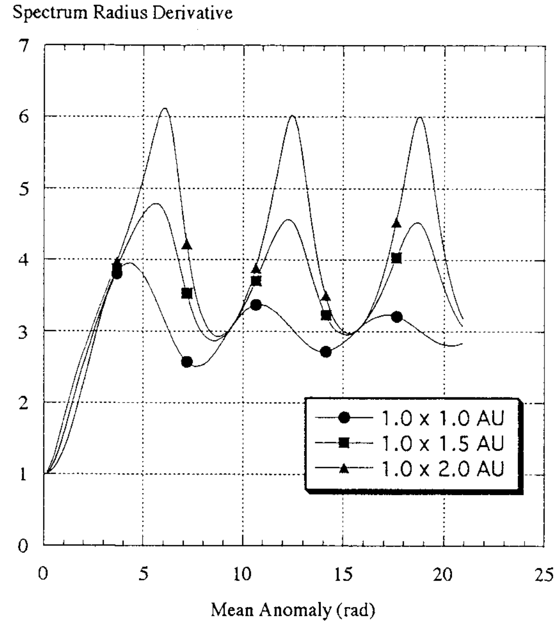

Fig. 2 Time derivative of the spectrum radii of the sensitivity matrices.

Two types of figures are shown for each case, the first showing the electrical acceleration magnitude a_{ep} and the second showing the electrical correction error $\delta\phi$. Three spacing strategies are evaluated: A) ordinary spacing law Eq. (18), B) spacing governed by navigation Eqs. (16) and (17), and C) spacing governed by electrical correction Eq. (15). In each simulation, the final correction point is automatically and uniquely determined as 5.26 days prior to the flyby according to the specified miss distance. With the parameter data from Table 3, ν and $2t/\delta\phi$ are calculated 21 and 16 days, respectively.

Figures 3 and 4 are obtained for the 10-day flight. The first correction epoch is fixed at 10 days prior to the flyby. Regardless of the a_{ep} and $\delta\phi$, the optimal second correction point degenerates to the initial point, 10 days prior to the arrival, since the flight period is short enough to satisfy the conditions for strategy B. The results are compatible with the earlier discussion. Figures 5 and 6 are shown for the 30-day flight. The first correction point is again fixed at the starting point. In this case, the results indicate dependency on a_{ep} and $\delta\phi$. As the a_{ep} and $\delta\phi$ get larger, the optimal second correction point gradually coincides with the point obtained from Eq. (15) in strategy C. In the case where a_{ep} and $\delta\phi$ are small, however, the results are close to the results indicated by strategy B. Figures 7 and 8 show the results for the case with a longer flight period, 100 days. As observed, the resulting optimal second correction points

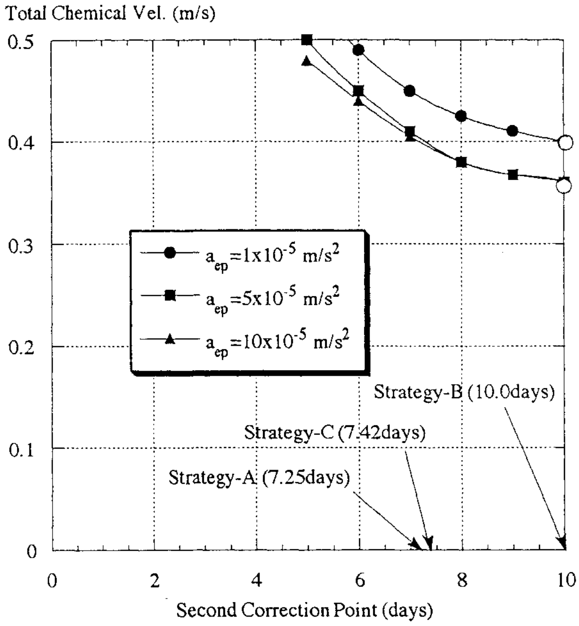

Fig. 3 Optimal correction point in terms of acceleration, 10-day flight.

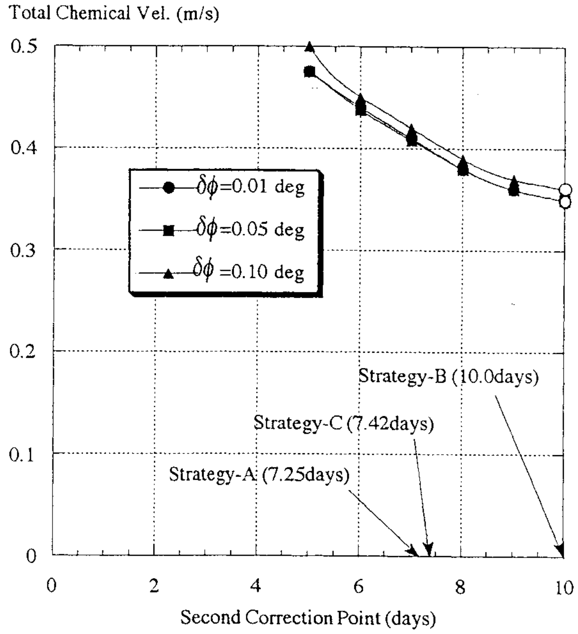

Fig. 4 Optimal correction point in terms of correction error, 10-day flight.

are compatible with those obtained via strategy C, regardless of the acceleration magnitude and the electric correction error $\delta\phi$. It should be stressed that strategy A never provides an optimal point in cases where electric propulsion is used. The usual interplanetary flight period in an electric propulsion case is much longer than 100 days, and it is obvious that the spacing rule is governed by strategy C, Eq. (15), which is independent of the statistical parameters, one of the major conclusions in this paper.

The preceding discussion puts forth Eq. (25) as an analytical evaluation of the chemical correction amount when the number of corrections is increased. Table 4 summarizes the chemical correction velocity increment in comparison with that obtained with Eq. (25). The results are obtained statistically with 100,000 Monte Carlo simulations. In this case, a flight period of 100 days is assumed and the specified miss distance is 150 km at flyby. Table 4 also lists the terminal miss distance in the last column and indicates that it is statistically kept within the specification, as expected. The chemical correction amount, characterized by the velocity increment, gradually approaches the actual figure as the number of corrections increases. Note that η is identical to unity (1.0) in this gravity-free field model and that Eq. (25) may well give a good estimate. As for the confidence verification in these Monte Carlo simulations, one statistical example (Figs. 9 and 10) shows the probability distribution plots for the case of the 100-day flight period with three corrections. Figure 9 indicates that the probability is 72% when the required correction velocity is below 3.71 m/s. Figure 10 shows that the probability is 77% when the terminal error is within 150 km,

Table 4 Chemical correction velocity increment and terminal miss distance

No. of corrections	Estimate via Eq. (25), m/s	Actual increment, m/s	Terminal miss distance, km
3	1.95	3.71	125
4	2.16	2.73	124
5	2.37	2.52	130
6	2.58	2.50	127

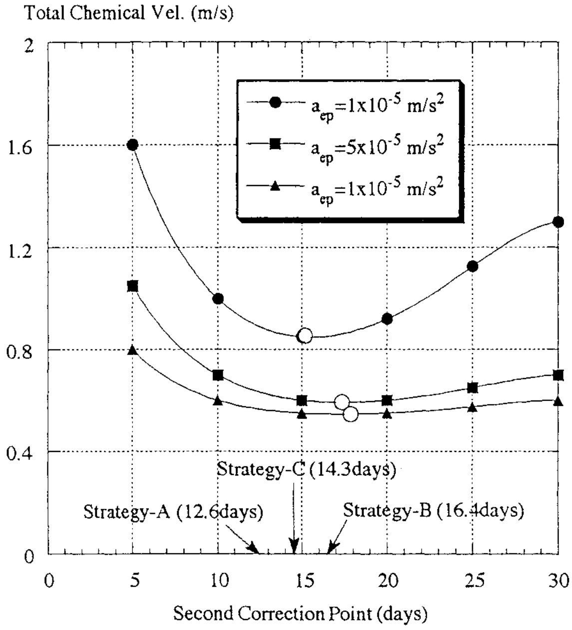

Fig. 5 Optimal correction point in terms of acceleration, 30-day flight.

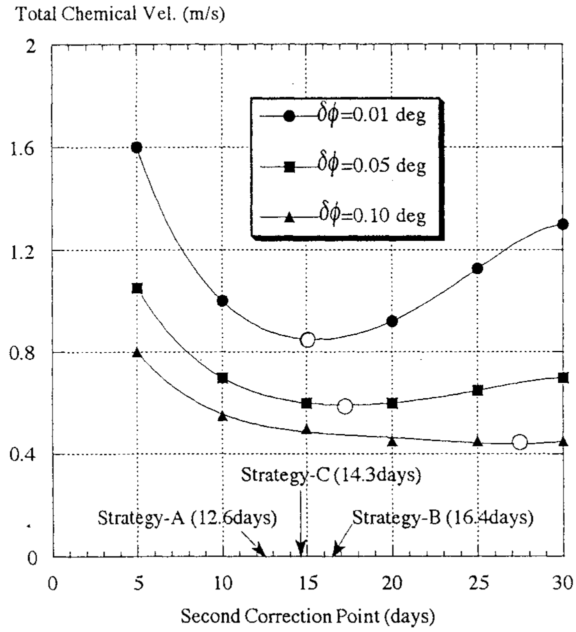

Fig. 6 Optimal correction point in terms of correction error, 30-day flight.

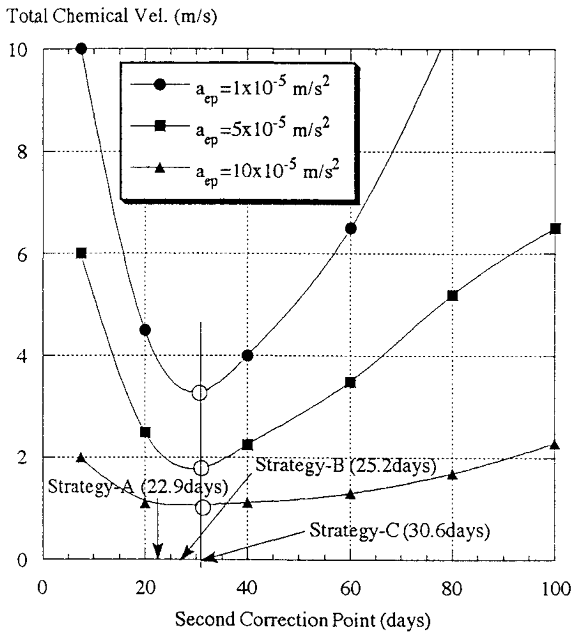

Fig. 7 Optimal correction point in terms of acceleration, 100-day flight.

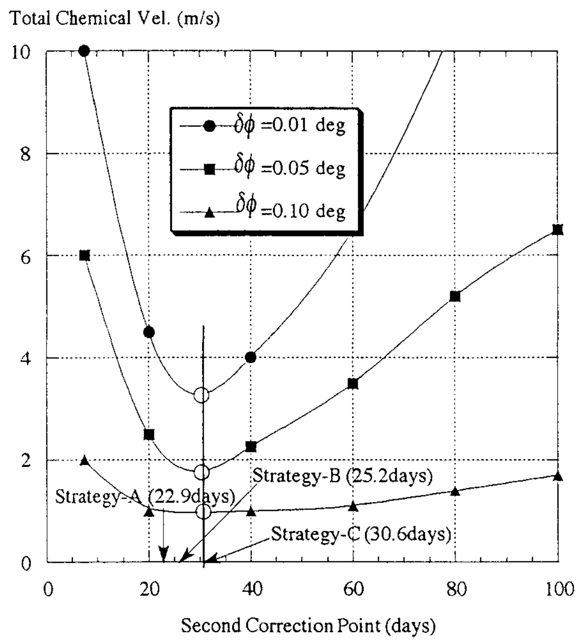

Fig. 8 Optimal correction point in terms of correction error, 100-day flight.

which is the specification. Both Figs. 9 and 10 indicate that the Monte Carlo simulations were correctly performed here.

The second example is one of the most realistic cases, a return path from asteroid Nereus (4660) to the Earth. The orbit plot is shown in Fig. 11. The assumed mission is included in the asteroid sample and return mission that is currently under investigation. The spacecraft assumed here is a small spacecraft that weighs approximately 300 kg when it is injected into the transasteroid orbit. In this mission, an ion thruster with 10-mN thrust is utilized, and so the electric power during the cruising phase can be dedicated to electric propulsion. According to the scenario, the departure from Nereus is planned on Oct. 27, 2003, arriving at Earth on Feb. 6, 2006. This is the flyby mission with the Earth, in which the spacecraft is supposed to re-enter into Earth's atmosphere to decelerate its orbital velocity, so that the soil sample can be recovered on the ground. Flyby speed with Earth is approximately 4.28 km/s. Total flight time is 833 days, and the total accelerated velocity increment via an ion thruster is approximately 3030 m/s. The following three distinct strategies were

Table 5 Chemical correction in the return path from asteroid

Strategy	First correction	Second correction	Third correction	Total velocity increment
1) Ordinary, days	230	63	17	—
m/s	527	70	0	597
2) New, days	350	105	17	—
m/s	165	69	54	288
3) Constant, days	561	289	17	—
m/s	2	3	533	538

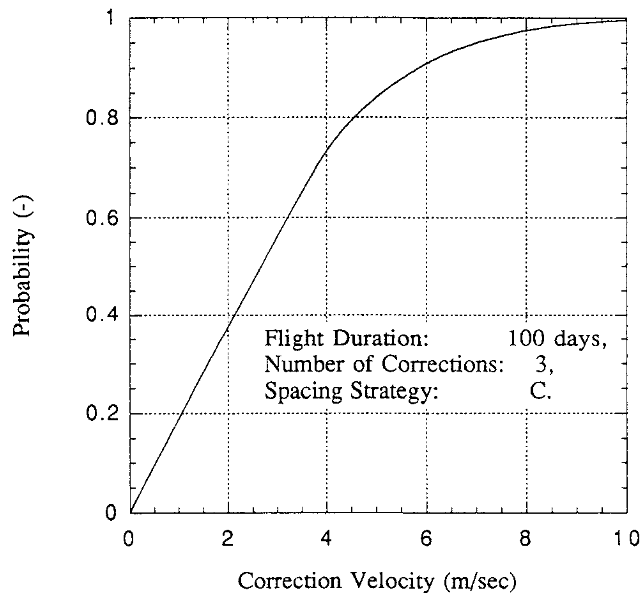

Fig. 9 Probability distribution for the correction velocity.

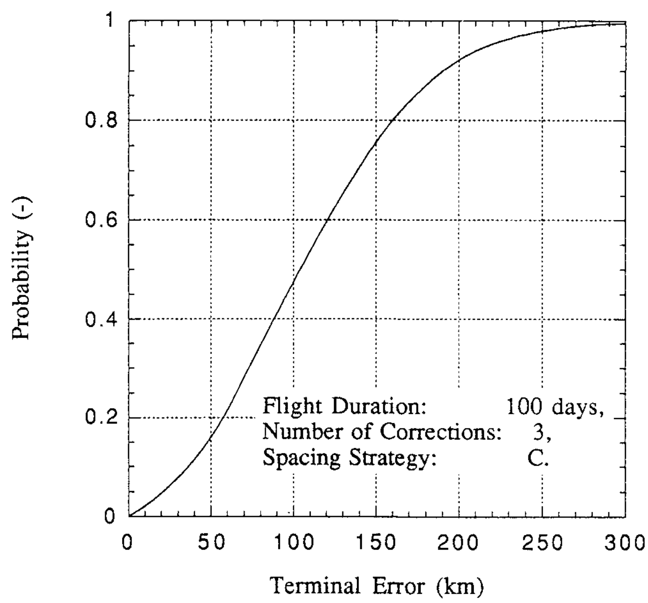

Fig. 10 Probability distribution for the terminal error.

applied to this example: 1) ordinary spacing law, 2) newly developed spacing law, and 3) constant interval spacing. Three midcourse corrections are assumed here for the demonstration. A propulsion efficiency degradation of 5% is introduced here, although neither navigation error nor impulsive correction error is incorporated. As noted, the final correction point is determined in view of the terminal miss distance and here it is set as 17 days prior to re-entry. The results obtained are summarized in Table 5.

The results show that the newly developed spacing law provides the minimum chemical correction velocity amount, which

Table 6 Five corrections in the return path from asteroid Nereus

	First correction	Second correction	Third correction	Forth correction	Fifth correction	Total ΔV
Time left, days	530	305	152	61	17	——
ΔV, m/s	4	73	0	43	5	125

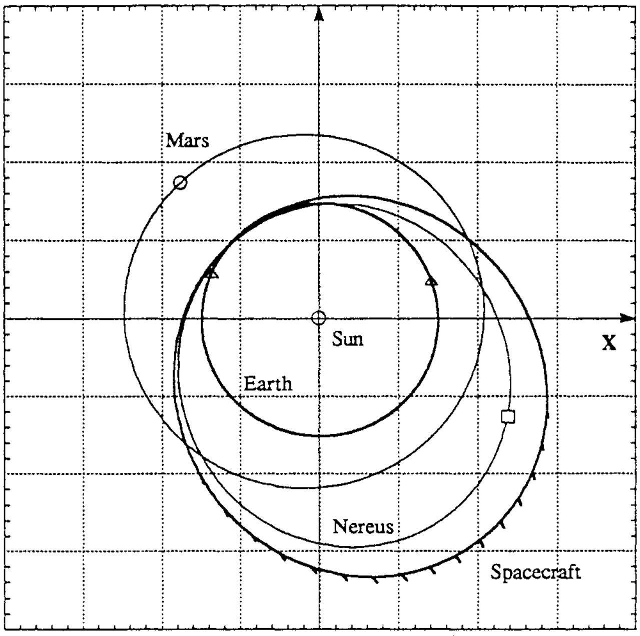

Fig. 11 Return path from Nereus (4660) to Earth.

has been confirmed in an actual planned mission scenario. The number of corrections has to be increased so that the total chemical correction amount can be reduced. Therefore, the results with five corrections are given in Table 6. The correction points were calculated according to Eq. (15). In Table 6 the total velocity increment is 125 m/s, whereas the second term Eq. (25) estimates $3030 \times 0.05 \times 530/833 = 96$ m/s. It can be stated that both the simulated and estimated ΔV agree with each other.

As pointed out in the discussion, the spacing rule is still valid even if the acceleration magnitude is varied along with the distance from the sun. In practical applications, the thrust level can be altered while the spacing rule is left unchanged.

Concluding Remarks

A new spacing rule is proposed that governs the midcourse correction points in missions using the electric propulsion. The discussion is generalized together with the navigation error and the impulsive correction error statistics. In addition, the analytical evaluation scheme for the chemical correction velocity increment is derived. Two numerical examples are shown here. They are 1) the flight in the idealized gravity free uniform field and 2) the return flight from asteroid Nereus to Earth. Both examples clearly demonstrate the discussion. The obtained results are expanded for 1) the case in which the propulsion thrust level is varied and 2) the case with coasting arcs that are given in the Appendices.

Appendix A: Generalized Spacing Rule in Case the Thrust Level is Altered

In some cases, the thrust level is altered depending on the power availability. The discussion here gives an extension of the previous discussion to the generalized form. The ultimate result shows that the new spacing rule Eq. (15) is still valid for such cases.

The electric correction error is expressed when the flight period left is divided in n subsegments:

$$\sum_{j=1}^{n}(a_{\mathrm{ep},j}\delta\phi_j\tau')\left\{T_k-\left(j-\frac{1}{2}\right)\tau'\right\}$$
$$= T_k\left(\sum_{j=1}^{n} a_{\mathrm{ep},j}\delta\phi_j\tau'\right) - \left[\sum_{j=1}^{n}\left(j-\frac{1}{2}\right)\tau'(a_{\mathrm{ep},j}\delta\phi_j\tau')\right]$$
$$= (T_k - T_k')S_k, \qquad \tau' = T_k/n \tag{A1}$$

where S_k is the total correction error and $S_k T_k'$ the first moment of the correction error. At the $(k+1)$th point, the chemical correction required is as follows:

$$\Delta V_{k+1} = \frac{1}{T_{k+1}}(\rho_k + \rho_{k+1}) + \varepsilon_{k+1} + \frac{T_k}{T_{k+1}}\lambda_k$$
$$+ \frac{\delta\phi_k}{T_{k+1}}\int_{T_{k+1}}^{T_k} t' a_{\mathrm{ep}}(t')\,\mathrm{d}t' - \frac{\phi_{k+1}}{T_{k+1}}(T_{k+1} - T_{k+1}')S_{k+1} \tag{A2}$$

Supposing that the electrical correction margin is exhausted, the total amount of the chemical correction is expressed as

$$\Delta V_{\mathrm{total}}' = \sum_{k=2}^{n}\left\{\frac{T_{k-1}}{T_k}\lambda_{k-1} + \frac{1}{T_k}(\rho_{k-1} + \rho_k)\right.$$
$$\left. + \frac{\delta\phi_{k-1}}{T_k}\int_{T_k}^{T_{k-1}} t' a_{\mathrm{ep}}(t')\,\mathrm{d}t'\right\} \tag{A3}$$

Note that Eq. (A3) consists only of the terms relating to the spacing rule. Assuming the statistical error parameters are constant and differentiating the preceding equation with respect to T_k, the following equation is obtained:

$$\frac{T_k}{T_{k+1}} = \frac{T_{k-1}}{T_k} + \frac{1}{T_k}\nu$$
$$+ \delta\phi\left\{\frac{T_k}{\tau_k} + \frac{1}{T_k}\int_{T_k}^{T_{k-1}} t'\frac{1}{\tau(t')}\,\mathrm{d}t' - \frac{T_k^2}{T_{k+1}}\frac{1}{\tau_k}\right\} \tag{A4}$$

The approximated spacing law is thus derived:

$$\frac{T_k}{T_{k+1}} \cong \frac{T_{k-1}}{T_k} + \frac{1}{T_k}\nu$$
$$+ \delta\phi\left\{\frac{T_k}{\tau_k} + \frac{1}{2\tau_k T_k}\left(T_{k-1}^2 - T_k^2\right) - \frac{T_k^2}{T_{k+1}}\frac{1}{\tau_k}\right\}$$
$$= \frac{T_{k-1}}{T_k} + \frac{1}{T_k}\nu + \frac{\delta\phi}{2\tau_k}T_k\left\{1 + \frac{T_{k-1}^2}{T_k^2} - 2\frac{T_k}{T_{k+1}}\right\} \tag{A5}$$

Equation (A5) corresponds to Eq. (14). Even in the case where τ_k is varied, if the last term in Eq. (A5) dominates, the same spacing rule as Eq. (15) is obtained:

$$\left(T_{k-1}^2/T_k^2\right) = 2(T_k/T_{k+1}) - 1 \tag{A6}$$

Appendix B: Extension to the Case with Coasting Period

Exhausted electric correction margin is not confined to the case in which the coasting period is filled up with powered arcs. If there is not enough electric propulsion, there will inevitably remain a coasting period with no correction margin. The new spacing rule is generalized to handle such cases.

Suppose l_k coasting arcs exist beyond the kth correction point, the anticipated terminal miss distance is expressed as follows:

$$\Delta x_{j,k+1} = \left\{\left(\frac{1}{2}a_{\mathrm{ep}}\delta\phi_k T_k^2 - \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k\sum_{j=1}^{l_k}\left(T_{cj}^2 - T_{cj}'^2\right)\right)\right.$$
$$\left. - \left(\frac{1}{2}a_{\mathrm{ep}}\delta\phi_k T_{k+1}^2 - \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k\sum_{j=1}^{l_{k+1}}\left(T_{cj}^2 - T_{cj}'^2\right)\right)\right\} + T_k\varepsilon_k + \rho_k$$
$$= \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k\left\{T_k^2 - T_{k+1}^2 - \sum_{j=l_{k+1}+1}^{l_k}\left(T_{cj}^2 - T_{cj}'^2\right)\right\} + T_k\varepsilon_k + \rho_k \tag{B1}$$

Here T_{cj} and T'_{cj} are the start end time of the jth coasting arc. The chemical correction amount may exceed the minimum fuel resulting from the navigation error as

$$\frac{1}{2}a_{\mathrm{ep}}\phi_k T_{k+1}^2 - \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k \sum_{j=1}^{l_{k+1}} \left(T_{cj}^2 - T_{cj}'^2\right) + T_{k+1}\Delta V_{k+1}$$

$$= \frac{1}{2}a_{\mathrm{ep}}\delta\phi_k \left\{ T_k^2 - T_{k+1}^2 - \sum_{j=l_{k+1}+1}^{l_k} \left(T_{cj}^2 - T_{cj}'^2\right) \right\}$$

$$+ T_k\varepsilon_k + \rho_k + T_{k+1}\varepsilon_{k+1} + \rho_{k+1} \tag{B2}$$

and the total chemical fuel amount throughout the flight is obtained as follows:

$$\Delta V_{\mathrm{total}} = \sum_{k=2}^{n} \left\{ \frac{T_{k-1}}{T_k}\lambda_{k-1} + \frac{1}{T_k}(\rho_{k-1} + \rho_k) + \frac{1}{2}a_{\mathrm{ep}}\delta\phi_{k-1} \frac{T_{k-1}^2 - T_k^2 - \sum_{j=l_k+1}^{l_{k-1}}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k} \right\}$$

$$+ \Delta V_1 - \sum_{k=1}^{n} \frac{1}{2}a_{\mathrm{ep}}\phi_k \frac{T_k^2 - \sum_{j=1}^{l_k}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k} \tag{B3}$$

When the electric correction margin is exhausted, the following relation holds:

$$\phi_0 \frac{T_0^2 - \sum_{j=1}^{l_0}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_0} = \sum_{k=1}^{n} \phi_k \frac{T_k^2 - \sum_{j=1}^{l_k}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k} \tag{B4}$$

As a result, Eq. (B3) is rewritten as

$$\Delta V_{\mathrm{total}} = \sum_{k=2}^{n} \left\{ \frac{T_{k-1}}{T_k}\lambda_{k-1} + \frac{1}{T_k}(\rho_{k-1} + \rho_k) + \frac{1}{2}a_{\mathrm{ep}}\delta\phi_{k-1} \frac{T_{k-1}^2 - T_k^2 - \sum_{j=l_k+1}^{l_{k-1}}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k} \right\}$$

$$+ \Delta V_1 - \frac{1}{2}a_{\mathrm{ep}}\phi_0 \frac{T_0^2 - \sum_{j=1}^{l_0}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_0} \tag{B5}$$

Assuming that the error parameters are constant, differentiating Eq. (B5) with respect to T_k gives the generalized spacing rule as follows:

$$\frac{T_k}{T_{k+1}} = \frac{T_{k-1}}{T_k} + \frac{1}{T_k}\nu + \frac{\delta\phi}{2\tau}T_k \times \left\{ 1 + \frac{T_{k-1}^2}{T_k^2} - \frac{\sum_{j=l_k+1}^{l_{k-1}}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k^2} - 2\frac{T_k}{T_{k+1}} \right\} \tag{B6}$$

This expression is the extension of Eq. (14) and versatile for use with any electrically propelled mission. Throughout almost the entire cruising flight, the rule degenerates to the following equation:

$$\frac{T_{k-1}^2 - \sum_{j=l_k+1}^{l_{k-1}}\left(T_{cj}^2 - T_{cj}'^2\right)}{T_k^2} = 2\frac{T_k}{T_{k+1}} - 1 \tag{B7}$$

In combination with the result discussed in Appendix A, Eq. (B7) is the most generalized spacing rule.

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